USGS
science for a changing world

Reciprocal Effects in Models: Overview

Jim Grace

U.S. Department of the Interior
U.S. Geological Survey

1

This module provides a general overview for the situation where two variables affect each other, aka reciprocal interactions. The is a type of nonrecursive model. Another type of model architecture related to this is “causal loops”.

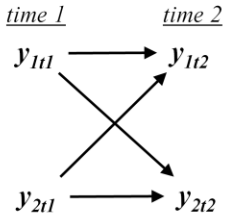
A general citation for this material is

Grace, J.B. 2006. Structural Equation Modeling and Natural Systems. Cambridge University Press. Cambridge, UK.

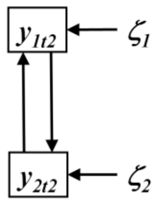
Notes: IP-56512; Support provided by the USGS Climate & Land Use R&D and Ecosystems Programs. Formal review of the material from which this tutorial was derived was provided by Jesse Miller and Phil Hahn, Univ. Wisconsin. Any use of trade, form, or product names is for descriptive purposes only and does not imply endorsement by the U.S. Government. Questions about this material can be sent to sem@usgs.gov. Last revised 15.06.17.

The causal diagram* allows us to treat time in more explicit fashion and can help us understand models with causal loops.

causal diagram* of reciprocal effects



static model of situation



The defining causal assumption is that manipulation of y_1 can lead to change in y_2 and vice versa.

2

Models containing reciprocal effects date back to the first models in econometrics, which dealt with supply and demand problems. In the field of economics, the solution of these types of models are referred to as “simultaneous equations modeling” (Wooldridge 2009).

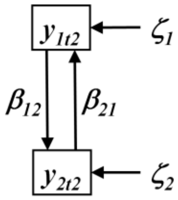
*Note that Pearl’s original description of the “causal diagram” does not deal explicitly with multiple time steps. There are several alternative approaches to graphical analysis, such as chain graphs and mixed graphs, that deal with such additional complexities. For simplicity and for the time being, I am simply calling this a causal diagram (sensu lato).

First Note: Unspecified causes of variations (aka U variables) that are understood to affect all four of the variables in the causal diagram on the left are not shown here.

Second Note: In the static model of the situation, the ζ (zetas) represent residuals.

Wooldridge, J.M. 2009. *Introductory Econometrics*. South-Western Publ. – see Chapter 16. Simultaneous Equations Models.

Houston, we have a problem! – Local non-identification



$$S = \begin{bmatrix} var1 & cov12 \\ cov12 & var2 \end{bmatrix}$$

$$\hat{E} = \begin{bmatrix} \zeta_1 & \beta_{21} \\ \beta_{12} & \zeta_2 \end{bmatrix}$$

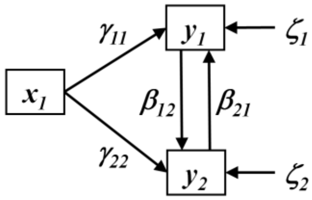
Observed covariance matrix S only has three unique elements, while the model-implied matrix Ê has four unique parameters.

USGS

3

Many people's first reaction to models with reciprocal effects is "Hey, that can be sorted out!" That reaction comes from the fact that people are thinking of the isolated case where there is only one covariance and you are trying to estimate two parameters. This is sort of equivalent of having two suspects for a crime and the only information is their testimony. As shown in the slides that follow, more information/evidence can indeed permit some estimation of the multiple effects if certain requirements are met.

Inadequate specification: (1) unidentified and (2) non-unique equations.



$$y_1 = \gamma_{11}x_1 + \beta_{21}y_2 + \zeta_1$$

$$y_2 = \gamma_{22}x_1 + \beta_{12}y_1 + \zeta_2$$

nonunique equations

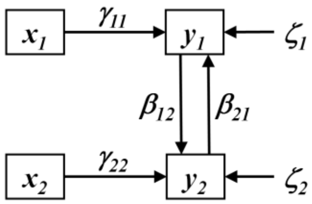
USGS

4

If we bring in one more piece of information, we now have three known bits, which are the observed covariances among the three variables. However, our model contains four parameters that must be estimated, so we still have an unidentified model.

There is a second problem here as well; we don't have any unique predictors for either y_1 or y_2. The next slide shows a solution to these problems.

Minimal requirements for model identification: (1) sufficient information and (2) unique predictors for the reciprocating variables.



$$y_1 = \gamma_{11}x_1 + \beta_{21}y_2 + \zeta_1$$

$$y_2 = \gamma_{22}x_2 + \beta_{12}y_1 + \zeta_2$$

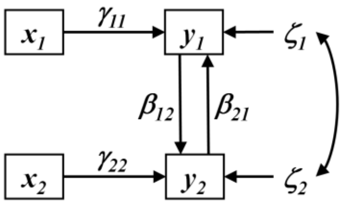
Unique equations

USGS

5

For identification, it is recognized that a minimum requirement is unique equations for each endogenous variable. This generally means some unique predictor for at least one of the variables involved in the reciprocal effect (a unique predictor for each, as shown here, is even better).

For models with reciprocal effects, we assume, until we find otherwise, that the errors for the reciprocating variables may be correlated.

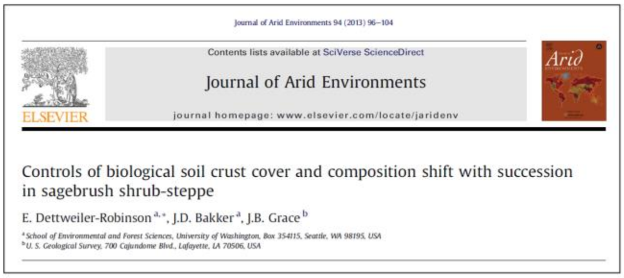


In a paper by

Wong, C.S. and Law, K.S. 1999. Testing reciprocal relations by nonrecursive structural equation models using cross-sectional data. *Organizational Research Methods* 2:69-87,

the authors explore a number of factors that influence the ability to extract proper estimates in models such as these. Among the things they show is that when there are correlated errors among the endogenous variables, as indicated here, one should include the error correlation as an estimable parameter in the model to obtain valid estimates.

Example



Journal of Arid Environments 94 (2013) 96–104

Contents lists available at SciVerse ScienceDirect

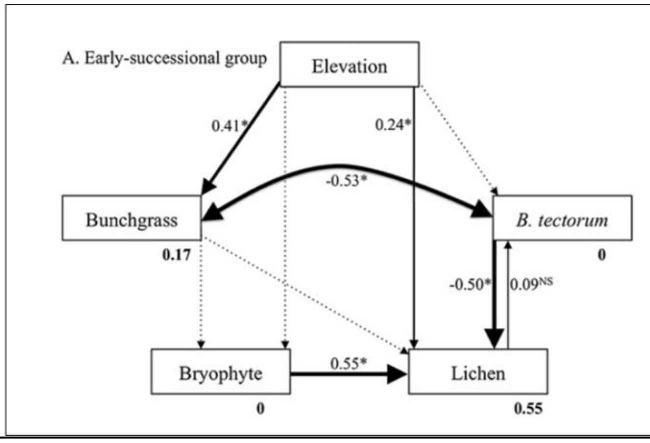
Journal of Arid Environments

journal homepage: www.elsevier.com/locate/jaridenv

Controls of biological soil crust cover and composition shift with succession in sagebrush shrub-steppe

E. Dettweiler-Robinson [a, *], J.D. Bakker [a], J.B. Grace [b]

[a] School of Environmental and Forest Sciences, University of Washington, Box 354115, Seattle, WA 98195, USA
[b] U. S. Geological Survey, 700 Cajundome Blvd., Lafayette, LA 70506, USA



USGS

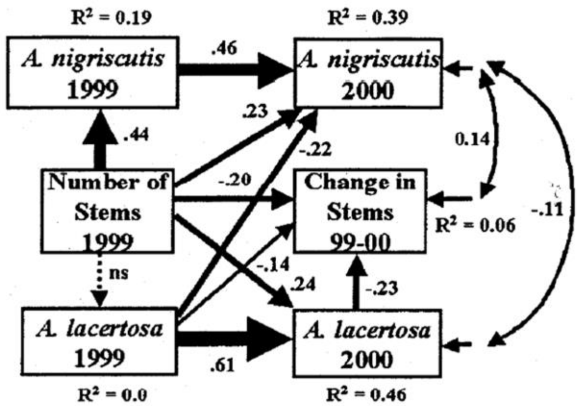
7

Structural equation models with reciprocal effects are reasonably common in the literature. As stated earlier, the general problem is central to economic models and one that cannot be ignored, in general. An introductory treatment is provided in Kline, R.B. 2006. Reverse arrow dynamics. Chapter 3 in Structural Equation Modeling: A Second Course. (eds. Hancock, G.R. and Mueller, R.O.) Information Age Publishing. Greenwich, CT., USA.

Here I show a figure from a paper where we examined factors associated with cheatgrass (*Bromus tectorum*) abundance. This particular model is somewhat incomplete in that it would have been nice to model both the interaction between *B. tectorum* and Lichen as well as between Bunchgrass and B. tectorum. In this model we adopted a conservative approach and represented that latter relationship via a negative error correlation (that probably stands in for a negative effect of Bunchgrass on *B. tectorum*). The focus in this case was on the potential for reciprocal effects between *B. tectorum* and Lichen and the results supported the conclusion of a uni-directional effect.

Caution! Our collapse of time requires assumption of equilibrium.

We would prefer, generally, a multi-time-step model.



USGS

Larson and Grace 2004. *Biological Control* 29:207-214. 8

When faced with modeling reciprocal effects, it is good to always keep in mind that such models require more stringent than usual assumptions, such as equilibrium. When possible, we are better off not collapsing time, as is the case here for a two time-step model of the interactions between a weed, Leafy Spurge, and the two biocontrol weavils that are used to control the weed (A. nigriscutis and A. lacertosa).

That said, one should be comfortable with the general requirements for models with reciprocal effects as there are many times we may need to go there rather than try to ignore these more complex causal situation.

More information can be found at http://www.nwrc.usgs.gov/SEM

USGS

I hope this overview has been useful. For more information, go to our webpage or search for examples involving your subject of interest. Questions and comments can be sent to sem@usgs.gov. Please note I cannot guarantee responses to individual inquiries, but will definitely incorporate suggestions in future tutorials. – Thanks!