science for a changing world

SEM Essentials: Interpreting Path Coefficients and Their Generalizations

Jim Grace

U.S. Department of the Interior
U.S. Geological Survey

This module seeks to illustrate certain basic ideas related to SEM through a description of "path rules". These rules were developed by Sewall Wright many years ago, but still represent some fundamental ideas important for SEM practitioners to understand.

A citation for the general information included in this module is:

Grace, J.B. (2006) Structural Equation Modeling and Natural Systems. Cambridge University Press.

A citation for the material on range standardization is

Grace, JB and Bollen K.A. 2005. Interpreting the results from multiple regression and structural equation models. Bulletin of the Ecological Society of America. 86:283-295.

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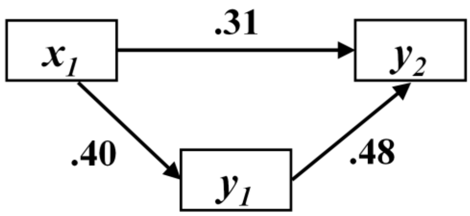
How should we interpret path coefficients? – As predictions!

The Concept of Statistical Control



The effect of y_1 on y_2 is controlled for the joint effects of x_1.

Recommend reading:

Grace, JB and Bollen K.A. 2005. Interpreting the results from multiple regression and structural equation models. Bulletin of the Ecological Society of America. 86:283-295.

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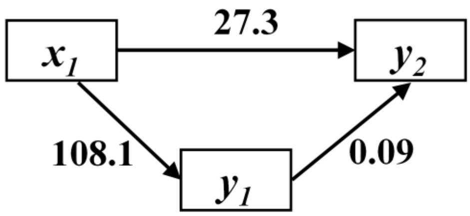
OK, we have been talking about coefficients, but how should we think about them? This is actually a subtle and complex topic and I plan to cover it in more depth in a separate module on "Interpreting Path Coefficients and Their Generalizations".

At the most basic level, the coefficients shown are predictions. They predict, for example, the effect on y2 if we were to vary y1 while holding the value of x1 constant.

Raw coefficients provide one kind of interpretation.

Raw coefficients.



For raw coefficients, the predicted effects are in raw units, such as the predicted change in milligrams biomass associated with a change in soil phosphorus concentration.

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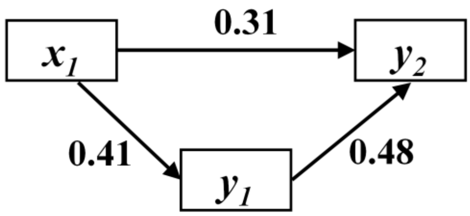
The direct interpretation of raw coefficients is pretty straightforward, at least in simple terms.

Standardized coefficients provide another sort of interpretation.

Standardized coefficients.



Let: γ_{x1y2} = a raw, unstandardized coefficient
r_{x1y2} = a conventional standardized coefficient
sd_{x1} and sd_{y2} = standard deviations of variables

Then, $r_{x1y2} = \gamma_{x1y2} * (sd_{x1} / sd_{y2})$

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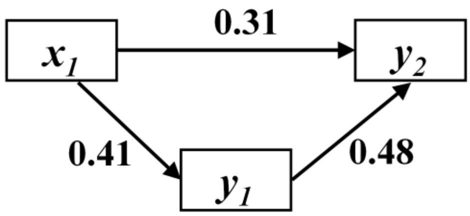
The interpretation of conventional standardized coefficients is more complex. They express the predicted changes in terms of standard deviation units. Note that the units for the raw coefficient in this case are are:

changes y2 as a function of changes in x1

and when multiplied by the ratio of standard deviations for x1/y2 causes the raw units to cancel out of the standardized coefficient.

Standardized coefficients are only interpretable within a sample.

Standardized coefficients.



The standardized coefficients are in the same units across various pathways. This implies comparability, something we often want.

However, standard deviations are estimated from variances and sample variances are influenced by a variety of things, making standardized coefficients non-transportable (i.e., non-generalizable).

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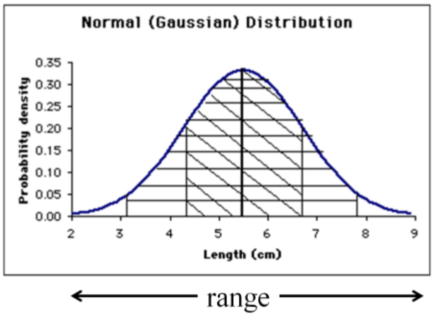
The assumptions that are associated with a clean interpretation of standardized coefficients have been questioned since Tukey raised this issue in 1954. Still today, while scientists often use standardized coefficients, many statisticians object to their use because they are sample specific and not generalizable.

Be aware that there is another related body of discussion that has gone on about the meaning of “importance”, which I am ignoring here for simplicity.

An alternative form of standardization* may be helpful.



6 standard deviations = 99% of the range for a true Gaussian distribution.

- improved interpretability
- better extrapolability because you can control comparisons over groups.

*Grace and Bollen (2005).

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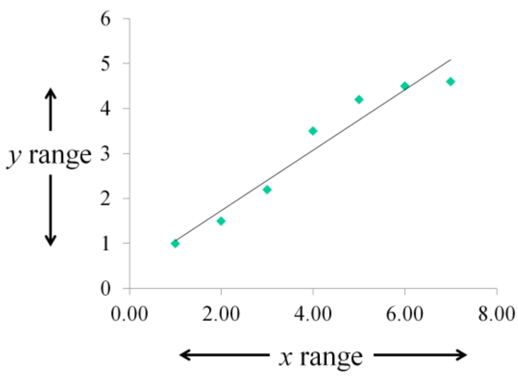
We can conceptualize standardized coefficients as attempting to express how a response variable will change across its range if we vary a predictor across its range.

I think the logic for using standard deviations to standardize is because they express the range of values in some fashion. For an idealized distribution, there is an approximate relationship between the range of values and the standard deviation.

Image from

http://www.rit.edu/~w-uphysi/uncertainties/Uncertaintiespart1.html

The “relevant range” standardization.



(1) Determine ranges and if you don’t like extreme values, prune your data.

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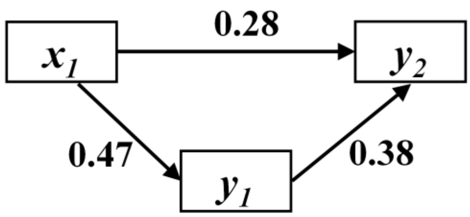
The “relevant range standardication method of Grace and Bollen (2005) represents a possible way to standardize in a more explicitly interpretable way. Of course estimating ranges is a sloppy business, even worse than estimation population standard deviations. So, we should not just automatically choose the computed range, but should consider the relationship.

Since we are making a prediction, it is helpful for us to make an explicit statement about the ranges over which we wish to generalize. We might even remove some data points if they hurt more than help our ability to generalize. If our theoretical knowledge permits extrapolation, for example to zero or one, then we may be able to justify such an extrapolation.

The "relevant range" standardization.

Range-standardized coefficients.



Let: γ_{x1y2} = a raw, unstandardized coefficient
q_{x1y2} = a range-standardized coefficient
rr_{x1} and rr_{y2} = relevant ranges of variables

Then, $q_{x1y2} = \gamma_{x1y2} * (rr_{x1} / rr_{y2})$

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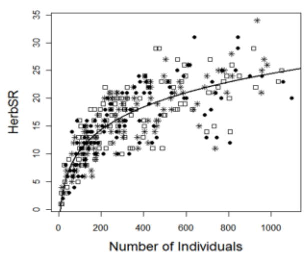
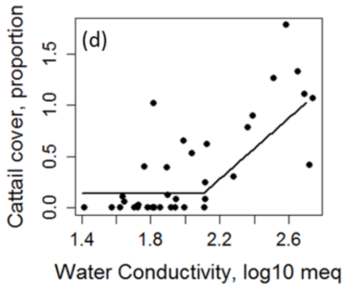
Relevant range coefficients will often be similar to regular standardized coefficients that are based on standard deviations. Perhaps the biggest advantage to using a relevant range approach is when comparing groups. When comparing coefficients across groups. regular standardization requires equal variance across groups, which is almost never the case. However, we may be able to establish a set of relevant ranges that apply across groups, thereby making the coefficients comparable across groups.

What about cases where relationships are not linear?





Here standard approaches don't work. Two general options, both of which will be illustrated elsewhere.
(1) Using prediction equations to run scenarios.
(2) Using composites to capture "signal strength" measures.

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For the more complex specification cases, the whole idea of summarizing linkages with traditional approaches becomes irrelevant.

One alternative approach builds off the range-standardization idea. One can use queries and ask how down-stream variables are predicted to change if we vary predictors across their observe ranges.

Another approach will be to use composites to estimate the strength of prediction for each link and then present coefficients that represent the strength of signal.

These possibilities are covered in more detail in other modules on

"Computing Quantities and Running Scenarios using Queries."

and

"Working with Composite Effects."

(note title approximate at this point).

Note also that there are several other issues of importance related to the interpretability of coefficients. In particular, whether and to what degree model parameters convey causal effects is also covered elsewhere in a separate module

"A Bit More about Causal Modeling Principles".