SEM Essentials: Anatomy of SE Models

Jim Grace

U.S. Department of the Interior
U.S. Geological Survey

In this module, I discuss a few points about the anatomy of models. The goal here is to introduce some terminology and to allow those not yet familiar with SEM to understand better what is being expressed in visual SEM presentations.

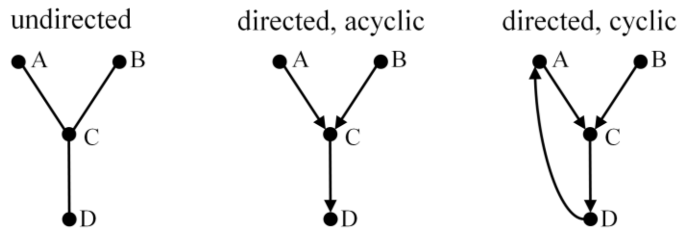
A citation that can be used for the information included in this module is:

Grace, J.B. (2006) Structural Equation Modeling and Natural Systems. Cambridge University Press

Notes: IP-056512; Support provided by the USGS Climate & Land Use R&D and Ecosystems Programs. I would like to acknowledge formal review of this material by Jesse Miller and Phil Hahn, University of Wisconsin. Many helpful informal comments have contributed to the final version of this presentation. The use of trade names is for descriptive purposes only and does not imply endorsement by the U.S. Government. Last revised 20141216. Questions about this material can be sent to sem@usgs.gov.

1. SEM falls within the broad field of graphical modeling, which is a fusion of probability theory and graph theory.

Types of graphs:



Graphical modeling (derived from graph theory), refers to
"nodes",
"edges", and
"pathways" along the edges.

We can analyze graphs for their causal logic.
We replace the nodes in graphs with variables when we develop SE models.

USGS

2

Here is an interesting quote articulating some of the reasons graphical models are fundamentally important.

"Graphical models are a marriage between probability theory and graph theory. They provide a natural tool for dealing with two problems that occur throughout applied mathematics and engineering -- uncertainty and complexity.

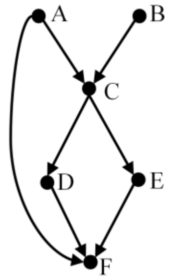
Many of the classical multivariate probabilistic systems studied in fields such as statistics, systems engineering, information theory, pattern recognition and statistical mechanics are special cases of the general graphical model formalism -- examples include mixture models, factor analysis, hidden Markov models, Kalman filters and Ising models. The graphical model framework provides a way to view all of these systems as instances of a common underlying formalism. This view has many advantages -- in particular, specialized techniques that have been developed in one field can be transferred between research communities and exploited more widely. Moreover, the graphical model formalism provides a natural framework for the design of new systems." --- Michael Jordan, 1998.

From http://www.cs.ubc.ca/~murphyk/Bayes/bnintro.html

2. Graphical modeling uses a different but overlapping terminology compared to traditional SEM literature.

Familial Relations:
parents vs. ***children***
ancestors vs. ***descendents***

Topological Relations:
collider nodes (C and F)
forks (C, D, E)
root nodes (A, B)
terminal nodes (F)



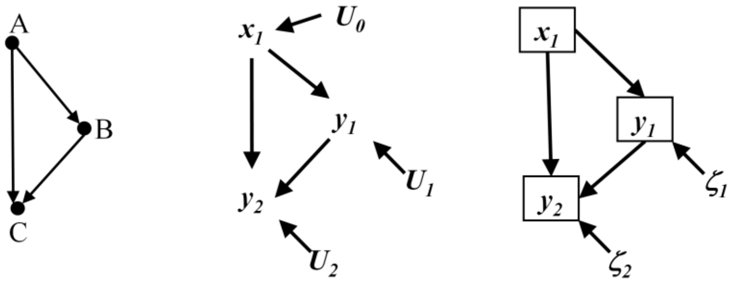
USGS

3

There is a wealth of terminology that has developed in different disciplines related to graphical models. Some of it is redundant across traditions. However, if you think about our science map and how different fields are disconnected to a large degree, it is not surprising. In this slide I show some of the terminology from the field of graphical modeling. It is worth knowing this additional terminology because in cases where one wishes to do formal causal analysis, such terms are essential.

3. We try to follow certain nomenclatural distinctions when representing graphs, causal diagrams, and models.

graph | causal diagram* | causal statistical model



Us are unspecified causal forces

ζ (zetas) are residuals

*A **causal diagram** is a graphical tool that enables the exploration of possible causal relationships between variables in a causal model.

USGS

4

In this slide I want to distinguish three different but overlapping conceptualizations of networks, “graphs”, “causal diagrams”, and “models”.

Graphs describe only general architectural features and are seldom used in SEM, except in teaching.

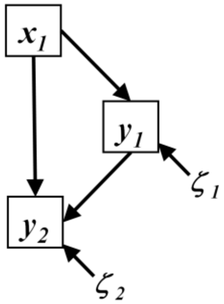
Causal diagrams, on the other hand, are a newer invention of Judea Pearl* (well, he actually invented them a while back, but we are only now seeing their clear value in SEM applications). Causal diagrams, in the narrow sense of that phrase, are actually a subset of the larger family of graphical models. They are meant to be a template of the system that can be used for deciding what data need to be collected to estimate particular effects. We can develop them without regard for what variables we actually have data for, which permit some deeper reasoning.

The “model” represents the relations among variables and errors of prediction.

*Pearl, J. 1995. Causal diagrams for empirical research. Biometrika 82:669–710.

4. Models are conceptualized in nonparametric, nonlinear terms (left), but commonly represented in simple linear form (right).

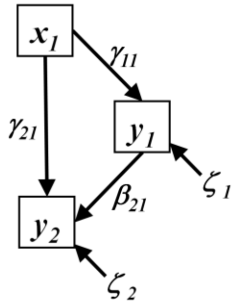
General (nonparametric) model representation



$y_1 = f(x_1)$
$y_2 = f(x_1, y_1)$

$\boxed{\mathbf{Y} = f(\mathbf{X}, \mathbf{Y})}$

Classical, simple linear model representation



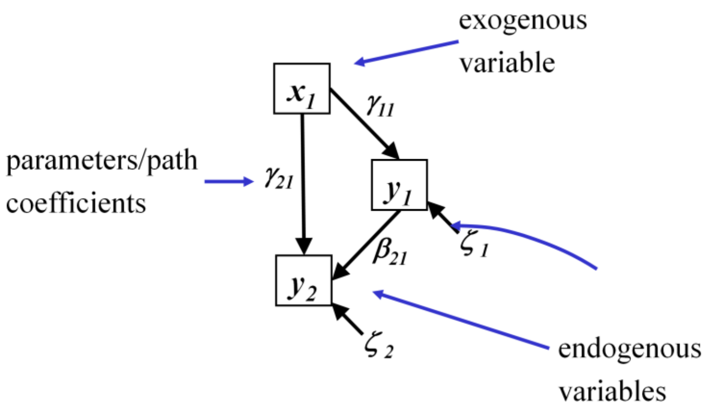
$y_1 = \alpha_1 + \gamma_{11}x_1 + \zeta_1$
$y_2 = \alpha_2 + \gamma_{21}x_1 + \beta_{21}y_1 + \zeta_2$

$\boxed{\mathbf{Y} = \alpha + \Gamma\mathbf{X} + \mathbf{B}\mathbf{Y} + \zeta}$

5

We often explain SEM using terminology made possible by simplified implementations such as Gaussian linear relations. We should not confuse this simplification with SEM itself, which is a framework for analysis and accommodates any statistical form.

5. Variables are classified as exogenous or endogenous for convenience.



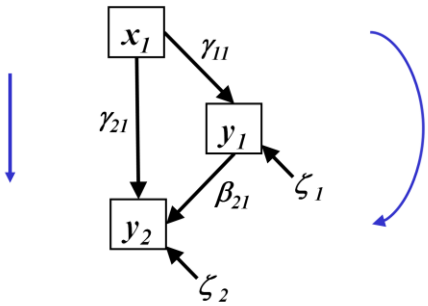
USGS

6

Exogenous variables have no arrows pointing at them (in models at least), endogenous do. Often parameter estimates are shown next to the arrows connecting variables, though not always. Here I use the Greek symbol for a small case gamma to represent links between exogenous and endogenous variables and betas to line endogenous variables. This is just common notation in many SEM implementations.

6. A key interest is partitioning relationships among pathways.

direct path from x_1 to y_2



indirect pathway from x_1 to y_2 through y_1

direct effect in the model = γ_{21} in this case.

indirect effect in the model = γ_{11} times β_{21} in this case.

total effect in the model = sum of direct and indirect effects.

USGS

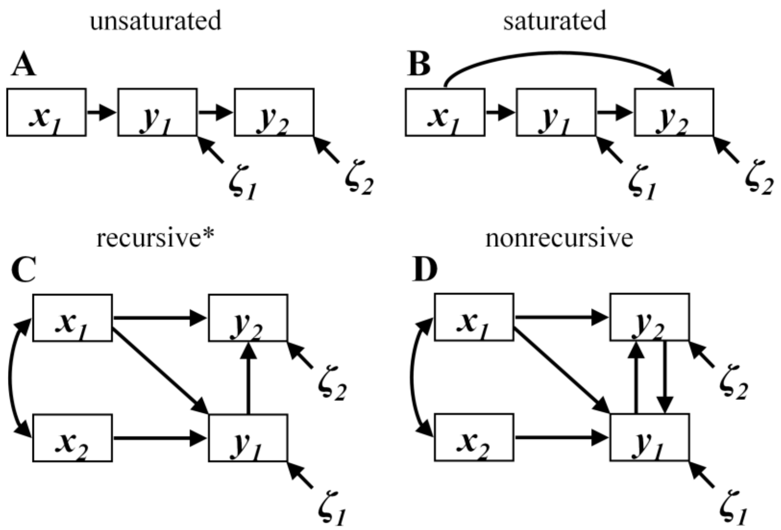
7

Each pathway tracing a route from a predictor to a response represents a distinct mechanism. In this slide I am trying to avoid a common misrepresentation related to the "effect" terminology (e.g., direct effect). That terminology causes lots of problems in presentations because it seems to confuse results with conclusions. It is helpful to be explicit that these are only "effects in the model", not in the real system. Two things are meant here by this statement.

(1) Caution must be taken when interpreting a direct effect in an SEM because in reality this "direct" effect is likely mediated by an unmeasured variable. In real systems, we assume that effects can be infinitely decomposed and ultimately operate at the quantum level. Thus, all true effects are indirect but since we are working with observed variables in our models, the effects actually specified in the models are simplified down to direct and indirect effects ("*in the model*").

(2) The conclusion that there is an "effect" depends on some causal assumptions that are not actually tested with the data. Thus, "effect in the model" clarifies that we recognize that critical assumption.

7. Several model architectures are possible.



*A **recursive** sequence is where each term is defined from earlier terms in the sequence.

8

Here is some terminology related to model architecture. Saturated models have links between all the variables (more strictly true, we have no model degrees of freedom). Saturated models represent a special class of model because they allow for everything to add up, meaning we can completely recover the observed matrix of covariances when our model is saturated. Unsaturated models have testable implications, however. Specifically, for Model A, the model hypothesizes that the indirect effect of x1 on y2 equals the observed covariance between those variables. This may not be empirically true, which would imply that there is some other connection between x1 and y2, such as in Model B.

Model C is "recursive" in that causation flows without loops. Model C also allows for x1 and x2 to be correlated for some unspecified reason. Nonrecursive models, such as in Model D, have more complex causal relationships. Here I show the case of a reciprocal interaction, which is a type of causal loop. To understand such models, we really need to consider the temporal dynamics that underlie the model, along with the assumptions required for collapsing time.